MA 441, Fall 2010

Assignment 8.

This homework is due *Tuesday* 11/23/2010.

There are total of 27 points in this assignment. 24 points is considered 100%. If you go over 24 points, you will get over 100% for this homework and it will count towards your course grade.

Collaboration is welcome. If you do collaborate, make sure to write/type your own paper.

This assignment covers sections 6.1 and 6.2.

(1) [3pt] (Exercise 6.1.4) Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^2$ for x rational, $f(x) = 0$ for x irrational. Show that f is differentiable at $x = 0$, and find $f'(0)$. (Hint: Use the limit definition of derivative.)

(2) (Part of exercise 6.1.8) Determine where each of the following functions from $\mathbb{R}$ to $\mathbb{R}$ is differentiable and find the derivative:
   (a) [3pt] $f(x) = x|x|$.
   (b) [3pt] $g(x) = 2x + |x|$.

(3) [3pt] (Exercise 6.1.9) Prove that if $f : \mathbb{R} \to \mathbb{R}$ is an even function and has a derivative at every point, then the derivative f' is an odd function. Also prove that if $g : \mathbb{R} \to \mathbb{R}$ is a differentiable odd function, then g' is an even function.

(4) [3pt] (Exercise 6.1.10) Let $g : \mathbb{R} \to \mathbb{R}$ be defined by $g(x) = x^2 \sin(1/x^2)$ for $x \neq 0$, and $g(0) = 0$. Show that g is differentiable for all $x \in \mathbb{R}$. Also show that the derivative g' is not bounded on the interval $[-1, 1]$.

(5) [3pt] (Exercise 6.2.3a) Find the points of relative extrema of the function $f(x) = |x^2 - 1|$ for $-4 \leq x \leq 4$.

(6) [3pt] (Exercise 6.2.6) Prove that $|\sin x - \sin y| \leq |x - y|$ for all $x, y \in \mathbb{R}$. (Hint: Apply the Mean Value theorem to sin on the interval $[x, y]$.)

(7) [3pt] (Exercise 6.2.8) Let $f : [a, b] \to \mathbb{R}$ be continuous on $[a, b]$ and differentiable on (a, b). Show that if $\lim_{x \to a} f'(x) = A$, then $f'(a)$ exists and is equal to A. (Hint: Use the limit definition of $f'(a)$ and apply the Mean Value Theorem to f on the interval $[a, x]$.)

(8) [3pt] (Exercise 6.2.17) Let f, g be differentiable on $\mathbb{R}$ and suppose that $f(0) = g(0)$, and $f'(x) \leq g'(x)$ for all $x \geq 0$. Show that $f(x) \leq g(x)$ for all $x \geq 0$. (Hint: Apply the Mean Value Theorem to $f - g$ on $[0, x]$.)